Paper Code : 21308

F–408

B. C. A. (Second Semester) EXAMINATION, 2019

(New Course)

Paper No. BCA—N—203

MATHEMATICS—II

Time : Three Hours] [*Maximum Marks : 70*

Note : Attempt any *five* questions. All questions carry equal marks. Symbols used are as usual.

1. (a) Prove that a bounded set has greatest and least limit points.

   (b) Prove that the derived set of every set is a closed set.

2. (a) If :

$$f(x) = \begin{cases} x+2 & \text{when} \quad n < 1 \\ 4x-1 & \text{when} \quad 1 \le n \le 3 \\ x^2+5 & \text{when} \quad n > 3 \end{cases}$$

find $\lim_{n \to 1} f(x)$ and $\lim_{n \to 3} f(x)$ if they exist.

   (b) Evaluate :

$$\lim_{x \to 0} \left[\frac{(1+x)^{1/x} - e}{x} \right].$$

3. (a) Check the continuity for $x = 0$ of f defined by :

$$f(x) = \frac{1}{1 - e^{1/x}}$$

when $x \neq 0$ and $f(0) = 0$.

   (b) Show that the sum, difference and product of two functions continuous at a point $x = a$ are continuous at this point.

4. (a) Test for convergence the series whose n^{th} term is :

$$\sqrt{n^3+1} + \sqrt{n^3-1}$$

   (b) Test for convergence and divergence of the series :

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 \ldots$$

5. State and prove p-series test.

6. (a) Expand $\sin x$ by Maclaurin's theorem.

   (b) If the mean value theorem is :

$$f(b) - f(a) = f'(x_1)(b-a)$$

find x_1 when $f(x) = x^3 - 3x - 1$, $a = -11/7$, $b = 13/7$.

7. (a) Find the maximum and minimum value of $(1-x)^2 e^x$.

(b) Show that the right circular cylinder of given surface (including the ends) and maximum volume is such that its height is equal to the diameter of the base.

8. (a) Show that if a sequence converges, its limit is unique.

(b) Check the roundedness and convergence of the sequence $z_1, z_2, z_3, \ldots, z_n, \ldots\ldots\ldots$ if $z_n = n \sin \frac{1}{2} n\pi$.

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