

Total No. of Questions : 9 | Total No. of Printed Pages : 4

Paper Code : 21304

F-404

B.C.A. (First Semester) Examination, 2021

(New Course)

Paper-No. BCA-104-N

MATHEMATICS-I

Time : Three Hours | Maximum Marks : 70

Note : Attempt any five questions. All questions carry equal marks. Symbols used have their usual meanings.

1. (a) If $x = \sin t$, $y = \sin p t$, prove that
$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0. \quad 7$$
- (b) If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0$. Hence find the value of y_n when $x=0$.
(1) P.T.O. 7

2. (a) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$ prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right).$$

7

- (b) If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u. \text{ and}$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}.$$

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3. (a) If $x+y=2e^0 \cos \phi$ and $x-y=2ie^0 \sin \phi$, show

$$\text{that } \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy \frac{\partial^2 u}{\partial x \partial y}. \quad 7$$

- (b) If $u^3 + v^3 = x + y$ and $u^2 v^2 = x^3 + y^3$, show that

$$\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2} \frac{y^2 - x^2}{uv(u - v)}. \quad 7$$

4. (a) Integrate : 7

$$\int \sin^4 x \cos^2 x \, dx.$$

(b) Evaluate :

7

$$\int_0^{\pi/2} \log \sin x \, dx.$$

5. (a) Find the volume of the solid obtained by revolving the cissoid $y^2(2a-x)=x^3$ about its asymptote.

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(b) Prove that $\text{div}(r^n R) = (n+3)r^n$. Hence show that R/r^3 is solenoidal.

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6. (a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

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(b) Solve $3x(1-x^2)y^2 \frac{dy}{dx} + (2x^2-1)y^3 = ax^3$.

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7. (a) Solve, by changing the independent

variable, $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} + 4x^3 y = x^5$.

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(b) Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sqrt{1 - \cos x}} = 1$ and show

that $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$.

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(3)

P.T.O.

8. (a) Find the value of 'a' if

$$f(x) = \begin{cases} 2x-1 & ; x < 2 \\ a & ; x = 2 \\ x+1 & ; x > 2 \end{cases}$$

7

(b) Find the length of the astroid :

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$$x^{2/3} + y^{2/3} = a^{2/3}.$$

9. (a) Find the vertex, focus and directrix of the parabola :

7

$$x^2 + 8x + 12y + 4 = 0.$$

(b) Find the equation of the hyperbola whose focus is (1, 1), directrix $2x+2y=1$ and eccentricity is $\sqrt{2}$.

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